STUDY GUIDE:

Module 6: Rational Numbers, Part 3

In this module we look at two other ways of expressing ratios. The first new way is based on the fact that we often like to measure in parts per hundred. A typical example of this is in grading papers where the grade is usually given as a *percent*. The symbol "%" is used to denote percent. "percent" literally means "per hundred". Hence when we ask, "How much is 23% of 800?" we mean, "At a rate of 23 per 100, how much will we take from 800?"

The problem with percents is that they are restricted to parts per 100. Thus, we can run into percents that can't be expressed as whole numbers. For example at a rate of 1 out of 3, you'd take 33 out of 99 and 34 out of 102. So at that rate you'd take more than 33 but less than 34 out of each hundred. Since 33% is too small to be $\frac{1}{3}$ and 34% is to great, we are compelled to invent numbers that measure amounts that are between whole numbers. The numbers we invent in this module are called *mixed numbers*. For example, we shall show in this module what it means to write that $\frac{1}{3}$ is $33\frac{1}{3}\%$.

Once we get used to the vocabulary we shall develop a simple way for doing arithmetic with percents and mixed numbers. Namely we shall translate all percents and mixed numbers to common fractions. We will then do the arithmetic in the language of common fractions (techniques we learned in Modules 4 and 5), and after we have the answers, we'll translate them back into the language of percents and mixed numbers.

Step 1:

View Videotape Lecture #6.

Step 2:

Read Module 6 of the text.

Step 3:

When you feel that you understand the material presented in Steps 1 and 2, complete the following "Check-The-Main-Ideas" self-quiz by correctly filling in each of the blanks.

Check The Main Ideas

"Percent" means "per _____". Since 3 X 25 = 75 and 4 X 25 = 100, at a rate of 3 out of each 4, you'd take ____ out of each 100. That is, $\frac{3}{4}$ = ____%. So if you got 75% on a test it means that you scored at a rate of 75 right answers out of each _____ questions. So if there were 200 questions on the test and you scored 75%, it means that you got _____ correct answers.

100 (hundred)

75; 75

100

150 (75 X 2)

Sometimes a percent won't be a whole number. For example 6 X 14 = 84 and 7 X 14 = 98. Hence at a rate of 6 out of each 7, you'd take 84 out of each _____. Therefore $\frac{6}{7}$ is _____ than 84%. Similarly, 6 X 15 = 90 and 7 X 15 = 105. Hence at a rate of 6 out of each 7, you'd take _____ out of each 105. Therefore $\frac{6}{7}$ is _____ than 90%.

98; more (greater)

90

less

To express $\frac{6}{7}$ exactly as a percent, we'd take $\frac{6}{7}$ of _____. To do this we'd multiply 100 by ___ and then divide this product by ___ to get 85 with a remainder of _____. In other words, $\frac{600}{7}$ = ___ + $\frac{5}{7}$. But rather than write 85 + $\frac{5}{7}$, we write $85\frac{5}{7}$, which is called a _____ number.

100

6; 7

5

85

mixed

Suppose, then, that there was a test that had 7 questions that were counted equally and you got 6 correct answers. Then your exact grade would be _____%. If the test score were rounded off to the nearest percent, your grade would be ____%.

$85\frac{5}{7}$

86

Any common fraction in which the numerator is at least as great as the denominator is called an _____ (common) fraction. To convert an improper fraction to a mixed number we divide the _____ of the fraction by the _____ of the fraction to get the whole number part of the mixed number and we then put the remainder over the _____ of the fraction to get the fractional part of the mixed number. For example, to write $\frac{149}{7}$ as a mixed number, we'd divide _____ by ____ to get 21 with a remainder of ____. So we'd place the 2 over ____ and conclude that $\frac{149}{7}$ is equivalent to the mixed number, _____.

improper

numerator; denominator

denominator

149, 7

2

7

$21\frac{2}{7}$

We reverse the process to convert a mixed number to an improper fraction. For example, if we had been given $21\frac{2}{7}$ and wanted to write it as a mixed number, we'd first multiply 21 by ____. We'd then add ___, and place the answer over ____. That is, $21\frac{2}{7} = \frac{(21 \times 7) + 2)}{7} =$ _____.

7

2; 7

$\frac{149}{7}$

We can do the arithmetic of mixed numbers by knowing the arithmetic of common fractions. For example suppose we wanted to multiply $8\frac{2}{3}$ by $1\frac{3}{5}$. Since (8 X 3) + 2 = 26, $8\frac{2}{3}$ is equivalent to the improper fraction _____; and since (1 X 5) + 3 = 8, $1\frac{3}{5}$ is equivalent to the improper fraction, _____.

$\frac{26}{3}$

$\frac{8}{5}$

Hence:

$$8\frac{2}{3} \times 1\frac{3}{5} = \frac{26}{3} \times \_\_\_\_$$

$\frac{8}{5}$

$$= \frac{26 \times 8}{3 \times 5}$$

So as an improper fraction in lowest terms,

$8\frac{2}{3} \times 1\frac{3}{5} =$ _____. To convert $\frac{208}{15}$ into a mixed number we divide _____ by ____ to get 13 with a remainder of _____. That is, $\frac{208}{15}$ is equivalent to the mixed number, _____.

$\frac{208}{15}$

208; 15

13

$13\frac{13}{15}$

Hence using only mixed numbers, we have that:

$8\frac{2}{3} \times 1\frac{3}{5} =$ _____.

$13\frac{13}{15}$

We can also convert a mixed number percent to a common fraction. For example, $7\frac{1}{3}\%$ means that you want $7\frac{1}{3}$ per 100. Since $7\frac{1}{3} \times 3 = 22$ and $100 \times 3 = 300$, a rate of $7\frac{1}{3}$ per 100 is the same as a rate of 22 per _____. In other words, $7\frac{1}{3}\%$ as a common fraction in lowest terms is _____. Another way to do this is to first convert $7\frac{1}{3}$ into the mixed number _____. Since "percent" means "per hundred" or "divided by 100" we annex ____ 0's to the denoinator of $\frac{22}{3}$ to get _____. Whichever way we do it, $7\frac{1}{3}\%$ names a rate of 22 per 300. Hence $7\frac{1}{3}\%$ of 600 is _____.

300

$\frac{22}{300}$

$\frac{22}{3}$

two

$\frac{22}{300}$

44 (22 X 2)

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Step 4:

Do the Mastery Review on the next page.

Mastery Review

ANSWERS:

1. How much is 83% of 400? 1. _____

2. A test consists of 400 true-false questions. A student answered 332 of these questions correctly. What percent of the questions did the student answer correctly? 2. _____

3. How much is 83% of 800? 3. _____

4. How much is 37% of 5,000?

5. Express 40% as a common fraction in lowest terms. 5. _____

6. How much is 40% of 5,000? 6. _____

7. Express 30% as an equivalent common fraction in lowest terms. 7. _____

8. How much is 30% of 5,000? 8. _____

9. Write $4\frac{1}{3}$ as a common fraction in lowest terms. 9. _____

10. Write $6\frac{3}{4}$ as a common fraction in lowest terms. 10. _____

11. Write $16\frac{7}{9}$ as a common fraction in lowest terms. 11. _____

12. Find the product of $4\frac{1}{3}$ and 9. 12. _____

13. Write $4\frac{1}{3}\%$ as an equivalent common in lowest terms. 13. _____

14. Write $\frac{1}{6}$ as an equivalent percent. 14. _____

15. Write $\frac{347}{6}$ as a mixed number. 15. _____

16. Write $\frac{9}{14}$ as a percent. 16. _____

17. Write $\frac{5}{6}$ of 1% as a common fraction in lowest terms. 17. _____

18. How much is $\frac{9}{100}\%$ of 70,000? 18. _____

Mastery Review (cont)

ANSWERS:

19. Write $4\frac{2}{3} \times 9\frac{1}{2}$ as a mixed number. 19. _____

20. Write $7\frac{3}{4} \div 2\frac{1}{5}$ as a mixed number. 20. _____

21. Write $7\frac{2}{3} + 5\frac{3}{4}$ as a mixed number. 21. _____

22. Write $7\frac{2}{3} - 5\frac{3}{4}$ as a mixed number. 22. _____

Answers to Mastery Review:

1. 332 2. 83(%) 3. 664 4. 1,850 5. 2/5 6. 2,000

7. 3/10 8. 1,500 9. 13/3 10. 27/4 11. 151/9

12. 39 13. 13/300 14. $16\frac{2}{3}\%$ 15. $57\frac{5}{6}$ 16. $64\frac{2}{7}\%$

17. 1/120 18. 63 19. $44\frac{1}{3}$ 20. $3\frac{23}{44}$ 21. $13\frac{5}{12}$

22. $1\frac{11}{12}$

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Step 5:

Do Self-Test 6, Form A on the next page.

Note:

Most people use mathematics to solve problems that occur in the "real world". For most of us, the "real world" is dollars-and-cents. So in this module the Self-Tests introduce a heavier emphasis on word problems involving money. You may find that you have to read a problem several times before you understand what arithmetic has to be done in order to solve the problem. Make sure that you work hard on learning how to translate words into mathematical operations.

It may seem strange, but reading comprehension is as important as arithmetical skills when we are called upon to solve "real world" problems. In most cases the arithmetical skills can be done by a calculator, but you have to be able to decide what skills are involved in a given problem.

Self-Test 6, Form A

ANSWERS:

(In exercises 1 through 6, write each expression as a mixed number; but if the answer is less than 1, write it as a common fraction in lowest terms.)

1. $9\frac{1}{2} + 5\frac{2}{3} + 2\frac{3}{4}$

2. $9\frac{1}{2} \times 5\frac{2}{3} \times 2\frac{3}{4}$

3. (a) $(9\frac{1}{2} - 5\frac{2}{3}) - 2\frac{3}{4}$

   (b) $9\frac{1}{2} - (5\frac{2}{3} - 2\frac{3}{4})$

4. (a) $(9\frac{1}{2} \div 5\frac{2}{3}) \div 2\frac{3}{4}$

   (b) $9\frac{1}{2} \div (5\frac{2}{3} \div 2\frac{3}{4})$

5. 39% of $9\frac{2}{3}$

6. $\frac{3}{7}$% of 1,500

7. A certain organization requires that at least 35% of its members be women. If the organization has 55 members, what is the least number of women that can belong to it?

8. For every \$7,500 a business needs, you agree to supply \$4,500 of it.
   (a) What percent of the need are you supplying?
   (b) If the business needs \$65,000 how much must you supply?

9. You buy a radio that is being sold at 80% of its regular price. Later you sell it to a friend for 70% of the price you paid. If the regular price of the radio was \$150:
   (a) How much did you pay for it?
   (b) How much did your friend pay for it?
   (c) What percent of the regular price did your friend pay for the radio?

10. $2\frac{3}{4}$ pounds of apples cost \$1.76.
   (a) What is the cost per pound for the apples?
   (b) At this rate how much would $5\frac{1}{2}$ pounds of apples cost?

ANSWERS:

1. _____
2. _____
3. (a) _____
   (b) _____
4. (a) _____
   (b) _____
5. _____
6. _____
7. _____
8. (a) _____
   (b) _____
9. (a) _____
   (b) _____
   (c) _____
10. (a) _____
   (b) _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 6, Form A

1. $17\frac{11}{12}$
2. $148\frac{1}{24}$
3. (a) $1\frac{1}{12}$ (b) $6\frac{7}{12}$
4. (a) $\frac{114}{187}$ (b) $4\frac{83}{136}$
5. $3\frac{77}{100}$
6. $6\frac{3}{7}$
7. 20
8. (a) 60% (b) \$39,000
9. (a) \$120 (b) \$84 (c) 56%
10. (a) 64¢ (\$0.64) per pound (b) \$3.52 (352 cents)

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If you did each problem in Form A correctly, you may, if you wish, proceed to the next module. Otherwise continue with Step 6.

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Step 6:

Study the solutions to Self-Test 6, Form A on the following pages, giving special attention to any problems you failed to answer correctly.

Solutions for Self-Test 6, Form A

From this point on we assume that you are familiar with the arithmetic of common fractions. Should you need additional review as we apply the results to Self-Test 6, restudy the applicable portions of Modules 4 and 5.

1.

The usual procedure for working with mixed numbers is to convert them to common fractions.

$$9\frac{1}{2} = \frac{(9 \times 2) + 1}{2} = \frac{19}{2}$$

$$5\frac{2}{3} = \frac{(5 \times 3) + 2}{3} = \frac{17}{3}$$

$$2\frac{3}{4} = \frac{(4 \times 2) + 3}{4} = \frac{11}{4}$$

Hence $9\frac{1}{2} + 5\frac{2}{3} + 2\frac{3}{4} = \frac{19}{2} + \frac{17}{3} + \frac{11}{4}$

Keep track of this procedure because we use it again and again. Namely, we translate the mixed number problem into a common fraction problem

We then use the results of Modules 4 and 5 to solve the resulting common fraction problem (remembering that the rules of arithmetic apply to both proper and improper common fractions).

Recognizing that 12 is the least common multiple of 2, 3, and 4, we have:

$$\frac{19}{2} = \frac{19 \times 6}{2 \times 6} = \frac{114}{12}$$

$$\frac{17}{3} = \frac{17 \times 4}{3 \times 4} = \frac{68}{12}$$

$$\frac{11}{4} = \frac{11 \times 3}{4 \times 3} = \frac{33}{12}$$

Then we take the answer in its common fraction form and translate it into an equivalent mixed number.

This mixed number is the answer to the problem.

So:

$$9\frac{1}{2} + 5\frac{2}{3} + 2\frac{3}{4} = \frac{114}{12} + \frac{68}{12} + \frac{33}{12}$$

$$= \frac{114 + 68 + 33}{12}$$

$$= \frac{215}{12}$$

and we now convert $\frac{215}{12}$ to a mixed number:

$$\begin{array}{r} 17\frac{11}{12} \\ 12\overline{)215} \\ -12 \\ \hline 95 \\ -84 \\ \hline 11 \end{array}$$

The remainder of 11 in this case means that we have 11 of the 12 (11/12) we need to have another "unit"

1. (cont)

Because a mixed number is the sum of a whole number and a proper common fraction and because addition is commutative and associative, we can solve this problem by a shorter method. Namely:

Method 2

$$9\tfrac{1}{2} + 5\tfrac{2}{3} + 2\tfrac{3}{4} =$$

$$(9 + \tfrac{1}{2}) + (5 + \tfrac{2}{3}) + (2 + \tfrac{3}{4}) =$$

$$(9 + 5 + 2) + (\tfrac{1}{2} + \tfrac{2}{3} + \tfrac{3}{4}) =$$

$$(9 + 5 + 2) + (\tfrac{6}{12} + \tfrac{8}{12} + \tfrac{9}{12}) =$$

$$16 + \tfrac{23}{12} =$$

$$16 + (1 + \tfrac{11}{12}) =$$

$$(16 + 1) + \tfrac{11}{12} =$$

$$17 + \tfrac{11}{12} =$$

$$17\tfrac{11}{12}$$

We can regroup only because all the operation symbols are plus signs.

$$12\overline{)23}\quad \begin{array}{l} 1 \\ -12 \\ \hline 11 \end{array}$$

R11 or $1\frac{11}{12}$ *or* $1 + \frac{11}{12}$

Don't leave the answer as $16\frac{23}{12}$. *Remember that in a mixed number, the fractional part has to be a proper fraction. That is, the numerator must be less than the denominator.*

In vertical form:

$$\begin{array}{r} 9\frac{1}{2} = 9\frac{6}{12} \\ 5\frac{2}{3} = 5\frac{8}{12} \\ \underline{2\frac{3}{4} = 2\frac{9}{12}} \\ 16\frac{23}{12} = \\ 17\frac{11}{12} \end{array}$$

As we shall see in the next few exercises, it is only with addition that we can work with the whole numbers and fractions separately.

Note that while Method 1 can always be used there are times when Method 2 is easier. For example it is easy to see that $26\frac{121}{307} + 23\frac{6}{307} = 49\frac{127}{307}$ but it would be a "mess" to rewrite the mixed numbers as equivalent improper fractions.

Unfortunately in most cases both methods can be tedious. This is one of the reasons we introduce decimal fractions in the next two modules.

2.

To simplify the "busy work" we've kept the same numbers in exercises 1 through 4. So from exercise 1 we already know that $9\frac{1}{2} = \frac{19}{2}$, $5\frac{2}{3} = \frac{17}{3}$, and $2\frac{3}{4} = \frac{11}{4}$.

Therefore:

$$9\tfrac{1}{2} \times 5\tfrac{2}{3} \times 2\tfrac{3}{4} = \frac{19}{2} \times \frac{17}{3} \times \frac{11}{4}$$

$$= \frac{19 \times 17 \times 11}{2 \times 3 \times 4}$$

$$= \frac{3,553}{24}$$

and we now translate $\frac{3,553}{24}$ into a mixed number:

$$\begin{array}{r} 148\text{R}1 \\ 24\overline{)3,553} \\ -2\,4 \\ \hline 1\,15 \\ -\ 96 \\ \hline 193 \\ -\ 192 \\ \hline 1 \end{array}$$

or $148 + \frac{1}{24}$ or $148\frac{1}{24}$

We could round off to see whether our answer is at least plausible. Namely

$9\frac{1}{2} \doteq 9$ *(or 10)*

$5\frac{2}{3} \doteq 6$

$2\frac{3}{4} \doteq 3$

Hence the product is approximately 9 X 6 X 3 = 162. In fact 10 X 6 X 3 = 180 must be too big because we replaced each mixed number by a greater whole number. So our answer should be in the vicinity of 180 but less than it.

Note that we could rewrite the problem as:

$(9 + \frac{1}{2}) \times (5 + \frac{2}{3}) + (2 + \frac{3}{4})$

but because multiplication and addition are involved in the same problem, we can't indiscriminately regroup and rearrange the terms. In fact, if we wanted to use only mixed numbers, we'd have to use the distributive property and the work would be quite complicated. Don't multiply the whole numbers to get 9 X 5 X 2 = 90 and the fractions to get $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$. $90\frac{1}{4}$ is much too small to be correct.

In fact, to use the distributive property, you'd have to add all possible combinations of a term from one set of parentheses by a term in any other set of parentheses

3.

(a)

$$(9\tfrac{1}{2} - 5\tfrac{2}{3}) - 2\tfrac{3}{4} =$$

$$(\tfrac{19}{2} - \tfrac{17}{3}) - \tfrac{11}{4} =$$

$$(\tfrac{114}{12} - \tfrac{68}{12}) - \tfrac{33}{12} =$$

$$\tfrac{46}{12} - \tfrac{33}{12} =$$

$$\begin{array}{r} 1\text{R}1 \\ 12\overline{)13} \\ -12 \\ \hline 1 \end{array} = 1\tfrac{1}{12}$$

(b)

$$9\tfrac{1}{2} - (5\tfrac{2}{3} - 2\tfrac{3}{4}) =$$

$$\tfrac{19}{2} - (\tfrac{17}{3} - \tfrac{11}{4}) =$$

$$\tfrac{114}{12} - (\tfrac{68}{12} - \tfrac{33}{12}) =$$

$$\tfrac{114}{12} - \tfrac{35}{12} =$$

$$\begin{array}{r} 6\text{R}7 \\ 12\overline{)79} \\ -72 \\ \hline 7 \end{array} = 6\tfrac{7}{12}$$

Parts (a) and (b) again emphasize the importance of grouping symbols. Without the parentheses both problems look like $9\frac{1}{2} - 5\frac{2}{3} - 2\frac{3}{4}$ *but the answers are different.*

Do the arithmetic within the parentheses first.

We've done all these steps before in Method 1 of exercise 1.

If you prefer to use the style of Method 2 of exercise 1, beware of how you "borrow". That is:

$$\begin{array}{rl} 9\frac{1}{2} & = 9\frac{3}{6} = 8\frac{9}{6} \\ -\ 5\frac{2}{3} & = 5\frac{4}{6} = 5\frac{4}{6} \\ \hline & \qquad 3\frac{5}{6} \end{array}$$

In terms of 6ths, when you "borrow" 1, you're borrowing 6 sixths, not 10 sixths.

$$\begin{array}{rl} 3\frac{5}{6} & = 3\frac{10}{12} \\ -\ 2\frac{3}{4} & = 2\frac{9}{12} \\ \hline & \qquad 1\frac{1}{12} \end{array}$$

4.

(a)

$$(9\tfrac{1}{2} \div 5\tfrac{2}{3}) \div 2\tfrac{3}{4} =$$

$$(\tfrac{19}{2} \div \tfrac{17}{3}) \div \tfrac{11}{4} =$$

$$(\tfrac{19}{2} \times \tfrac{3}{17}) \div \tfrac{11}{4} =$$

$$\tfrac{57}{34} \div \tfrac{11}{4} =$$

$$\tfrac{57}{34} \times \tfrac{4}{11} =$$

$$\frac{57 \times 4}{34 \times 11} =$$

$$\frac{57 \times \overset{2}{\cancel{4}}}{\underset{17}{\cancel{34}} \times 11} =$$

$$\frac{114}{187}$$

Just as in the previous exercise, notice the importance of the grouping. Again remember to the arithmetic inside the parentheses first.

Keep in mind we only need common denominators for addition and subtraction; not for multiplication or division.

Did you notice that once we converted the mixed numbers into common fractions, we used only the rules learned in Modules 4 and 5?

To my knowledge, there is no way of doing this problem without first converting the mixed numbers to common fractions.

114 = 57 X 2 = 3 X 19 X 2 and 187 = 17 X 11. Since the prime factorizations have no common factors, 114/187 is in lowest terms.

(b)

$$9\tfrac{1}{2} \div (5\tfrac{2}{3} \div 2\tfrac{3}{4}) =$$

$$\tfrac{19}{2} \div (\tfrac{17}{3} \div \tfrac{11}{4}) =$$

$$\tfrac{19}{2} \div (\tfrac{17}{3} \times \tfrac{4}{11}) =$$

$$\tfrac{19}{2} \div \tfrac{68}{33} =$$

$$\tfrac{19}{2} \times \tfrac{33}{68} =$$

$$\begin{array}{r} 4\text{R}83 \\ 136\overline{)627} \\ -544 \\ \hline 83 \end{array} \quad = 4\tfrac{83}{136}$$

Partial Check:

(4 X 136) + 83 =

544 + 83 =

627

Hence: $4\tfrac{83}{136} = \tfrac{627}{136}$

5.

Again we shall convert the problem into one that involves only common fractions.

39% means 39 per 100 or $\frac{39}{100}$

$9\frac{2}{3} = \frac{(9 \times 3) + 2}{3}$ or $\frac{29}{3}$

Hence:

39% of $9\frac{2}{3}$ =

$\frac{39}{100} \times \frac{29}{3}$ =

$\frac{39 \times 29}{100 \times 3}$ =

$\frac{\overset{13}{\cancel{39}} \times 29}{100 \times \underset{1}{\cancel{3}}}$ =

$\frac{13 \times 29}{100}$ =

$100\overline{)377}$ 3R77 = $3\frac{77}{100}$

-300

77

Remember that "of" means "X"

As an alternative method we could first find 1% of 9 2/3 and then multiply this by 39 to find 39% of 9 2/3. That is:

1% of $9\frac{2}{3} = 1\%$ *of* $\frac{29}{3}$.

$= \frac{1}{100} \times \frac{29}{3}$

$= \frac{29}{300}$

So 39% of $9\frac{2}{3} = 39 \times \frac{29}{300}$

$= \frac{39}{1} \times \frac{29}{300}$

which checks with the answer we got the other way. The main point is that while there is only one right answer there are often many right methods.

Plausibility Check

39% is a "little" less than 40% which is $\frac{40}{100}$ *or* $\frac{2}{5}$. $9\frac{2}{3}$ *is a "little" less than 10. Hence we should expect 39% of* $9\frac{2}{3}$ *to be a "little" less than* $\frac{2}{5}$ *of 10, which is 4. Certainly* $3\frac{77}{100}$ *qualifies as being a "little" less than 4.*

6.

$\frac{3}{7}\%$ means $\frac{3}{7}$ of 1% or $\frac{3}{7}$ X 1%, and since 1% means 1 per 100, we have:

$$\frac{3}{7}\% = \frac{3}{7} \times \frac{1}{100}$$

$$= \frac{3}{700}$$

Hence:

$\frac{3}{7}\%$ of 1,500 =

$\frac{3}{700}$ X 1,500 =

$\frac{3}{700} \times \frac{1,500}{1}$ =

$\frac{4,500}{700}$ =

$$7\overline{)45}\quad 6\text{R}3 = 6\frac{3}{7}$$
$$-42$$
$$3$$

The "quick" way of converting 3/7 % to a fraction is to delete the % symbol and annex two 0's to the denominator.

So $\frac{3}{7}\%$ is a rate of 3 per 700.

Obtaining an estimate:

$\frac{3}{7}$ is a bit less than $\frac{1}{2}$

1% of 1,500 = 1,500 ÷ 100

= 15

Hence:

$\frac{1}{2}\%$ of 1,500 = $\frac{1}{2}$ of 15 or $7\frac{1}{2}$

Therefore, $\frac{6}{7}\%$ of 1,500 should be a little less than $7\frac{1}{2}$. Hence, $6\frac{3}{7}$ is a reasonable answer (but 64 would not be reasonable).

Method 2:

$$\frac{3}{700} \text{ of } 1,500 = (1,500 \div 700) \times 3$$

$$= \frac{15}{7} \times 3$$

$$= (2 + \frac{1}{7}) \times 3$$

$$= (2 \times 3) + (\frac{1}{7} \times 3)$$

$$= 6 + \frac{3}{7}$$

Here we using the distributive property

Perhaps you've found other correct methods you like better. Don't be afraid to use any correct method you like. But if a method you like doesn't yield the right answer, check with someone to see why it doesn't work.

7.

Except that this exercise is the first application of percents in this module, the only new thing here is the meaning of "at least 35%".

The point is that the number of women must be at least 35% of the membership. Since there are 55 members, the number of women must be at least:

$$35\% \text{ of } 55 = \frac{35}{100} \times 55$$

$$= \frac{35}{100} \times \frac{55}{1}$$

$$= \frac{35 \times 55}{100}$$

$$= \frac{5 \times 7 \times 5 \times 11}{2 \times 2 \times 5 \times 5}$$

$$= \frac{77}{4} \quad 19\text{R}1 = 19\frac{1}{4}$$

$$4\overline{)77}$$

$$-4$$

$$37$$

$$-36$$

$$1$$

Remember that when we deal with "people" all answers must be whole numbers.

We're going to use prime factorization to reduce the common fraction to lowest terms, but we don't have to.

$$\frac{35 \times 55}{100} = \frac{1,925}{100} \quad 19\text{R}25$$

$$100\overline{)1,925}$$

$$-\ 1\ 00$$

$$925$$

$$-\ 900$$

$$25$$

So the organization must have at least $19\frac{1}{4}$ women. In particular, then, 19 women would be a trifle too small. Hence there must be at least 20 women in the organization.

Notice that 19 1/4 to the nearest whole number rounds off to 19. However, since we have to have at least 19 1/4 women, we round UP to 20.

As a quick check, 20 out of 55 is $\frac{20}{55}$ or $\frac{4}{11}$, and as a percent, we have:

$$\frac{4}{11} \text{ of } 100 = \frac{400}{11} \quad 36\text{R}4$$

$$11\overline{)400}$$

$$-33$$

$$70$$

$$-66$$

$$4$$

This is slightly more than 35%, but if we had 19 women we'd have less than 35%.

19 out of 55 is $(\frac{19}{55} \times 100)\%$ or:

$$\frac{1,900}{55} = \frac{380}{11} \quad 34\ R6$$

$$11\overline{)380}$$

$$-33$$

$$50$$

$$-44$$

$$6$$

8.

This is similar to problems we did in Module 5, except that now we are emphaszing the percent rather than the common fraction notation.

A rate of \$4,500 per \$7,500 is represented by the common fraction:

$$\frac{4,500}{7,500} = \frac{45}{75}$$

$$= \frac{9}{15}$$

$$= \frac{3}{5}$$

Make sure you understand how we're reducing the fraction.

(a) In terms of percent, a rate of 3 out of 5 is the same as a rate of 60 per 100. That is:

$$\frac{3}{5} = \frac{3 \text{ X } 20}{5 \text{ X } 20} = \frac{60}{100} = 60\%$$

Up to now this could have been an exercise in Self-Test 5.

From another point of view, we're taking 3/5 of 100.

So Part (a) simply involves translating 3/5 to 60%.

(b) What part (a) tells us is that you have agreed to pay for 60% of the company's financial needs. Since the need is \$65,000, you have agreed to pay:

$$60\% \text{ of } 65,000 = \frac{60}{100} \text{ of } 65,000$$

$$= 60 \text{ X } 650$$

$$= \$39,000$$

In other words, at a rate of \$3 per \$5, you supply \$39,000. That is:

$\frac{3}{5}$ *of* \$65,000 = (\$65,000 ÷ 5) *or* \$13,000 X 3

If we hadn't have done part (a) first and we didn't want to use percents, we could have computed:

$$\frac{4,500}{7,500} \text{ of } \$65,000$$

which reduces to

$$\frac{3}{5} \text{ of } \$65,000$$

9.

Except for the fact that we're using the language of percents, this problem simply involves taking a fractional part of a fractional part. Let's proceed part-by-part.

(a)

You're paying 80% of the regular price and the regular price is \$150. Hence you're paying:

$$80\% \text{ of } \$150 = \frac{80}{100} \times \$150$$

$$= \frac{4}{5} \text{ of } \$150$$

$$= 4 \times (\$150 \div 5)$$

$$= 4 \times \$30$$

$$= \$120$$

$\frac{80}{100} = \frac{4 \times 20}{5 \times 20}$

(b)

Your friend is paying 70% of what you paid for the radio. In part (a) we saw that you paid \$120 for the radio. Hence your friend is paying

$$70\% \text{ of } \$120 = \frac{70}{100} \text{ of } \$120$$

$$= \frac{7}{10} \text{ of } \$120$$

$$= 7 \times (\$120 \div 10)$$

$$= 7 \times \$12$$

$$= \$84$$

Be careful here. Your friend is paying 70% of \$120 not 70% of \$150. Read the problem carefully.

(c)

Based on the regular price, your friend is paying \$84 of the \$150. That is:

$$\frac{\$84}{\$150} = \frac{84}{150} = \frac{42}{75} = \frac{14}{25}$$

Again be sure to read carefully. If the problem was based on the price you paid the ratio would have been \$84 out of \$120.

9 (c). (cont)

So based on the regular price, your friend pays \$14 out of each \$25 or 56%. That is:

$$\frac{14}{25} = \frac{14 \times 4}{25 \times 4} = \frac{56}{100} = 56\%$$

Important Note:

Regardless of what you paid for the radio, your friend pays 56% of the regular price. The key point is that your friend is paying

70% of 80% of the regular price.

$$70\% \text{ of } 80\% = \frac{70}{100} \text{ of } \frac{80}{100}$$

$$= \frac{70 \times 80}{100 \times 100}$$

$$= \frac{5,600}{10,000}$$

$$= \frac{56}{100}$$

$$= 56\%$$

If we didn't know the regular price, we would still know that your friend would pay \$14 out of each \$25 of the regular price. That is:
\$14 out of \$25
\$28 out of \$50
\$42 out of \$75
\$56 out of \$100
\$70 out of \$125
\$84 out of \$150
\$98 out of \$175 and so on

10.

This problem is very similar to the ones in Module 5 where we found the price per pound using common fractions.

This is why it's so important to learn the previous modules. We're doing exactly what we've been doing before but simply in a different "language"

(a)

The label "cents per pound" suggests

cents ÷ pounds

\$1.76 means \$1 + 76 cents or 100 cents + 76 cents.

So we have:

$$176 \text{ cents} \div 2\frac{3}{4} \text{ pounds} = 176 \text{ cents} \div \frac{11}{4} \text{ pounds}$$

10 (a). (cont)

$= (176 \div \frac{11}{4})$ cents per pound

"cents per pound" still means "cents : pounds"

$= (176 \times \frac{4}{11})$ cents per pound

$= (\frac{176}{1} \times \frac{4}{11})$ cents per pound

From another point of view, we're taking 4/11 of 176.

$= (\frac{176 \times 4}{11})$ cents per pound

$= (\frac{\cancel{11} \times 16 \times 4}{\cancel{11}})$ cents per pound

= 64 cents ($0.64) per pound

As a rough check the price is approximately 3 pounds for $2 and this is about 67¢ per pound. So our answer is reasonable.

Check:

At 64 cents per pound, 1/4 of a pound would cost 1/4 of 64 cents or 16 cents. Hence:

2 pounds @ 64¢ = 128¢
3 fourths @ 16¢ = 48¢
$2\frac{3}{4}$ pounds @ 64¢ = 176¢ or $1.76

(b)

Since the price per pound is 64 cents, the price of $5\frac{1}{2}$ pounds will be 64 cents X $5\frac{1}{2}$ or:

64 cents X $\frac{11}{2}$ =

32 cents X 11 =

352 cents = $3.52

At 64¢ per pound, 5 pounds would cost 64 cents X 5 or $3.20. 6 pounds would cost 64 cents X 6 or $3.84. This tells us that the answer to part (b) must be between $3.20 and $3.84--which $3.52 is.

Note:

We can do this problem another way. Since each 2 3/4 pounds cost $1.76, we need only see how many times 2 3/4 "goes into" 5 1/2. $5\frac{1}{2} \div 2\frac{3}{4} = \frac{11}{2} \div \frac{11}{4} = \frac{11}{2} \times \frac{4}{11} = \frac{4}{2} = 2$. *So we pay $1.76 twice or $3.52*

In other words $5\frac{1}{2} = 2 \times 2\frac{3}{4}$. *So we're buying* $2\frac{3}{4}$ *pounds twice.*

****

Step 7:

Do Self-Test 6, Form B on the next page.

Self-Test 6, Form B

In exercises 1-6, write each answer as a mixed number.

1. $10\frac{1}{3} + 3\frac{1}{2} + 1\frac{3}{5}$

2. $10\frac{1}{3} \times 3\frac{1}{2} \times 1\frac{3}{5}$

3. (a) $(10\frac{1}{3} - 3\frac{1}{2}) - 1\frac{3}{5}$

   (b) $10\frac{1}{3} - (3\frac{1}{2} - 1\frac{3}{5})$

4. (a) $(10\frac{1}{3} \div 3\frac{1}{2}) \div 1\frac{3}{5}$

   (b) $10\frac{1}{3} \div (3\frac{1}{2} \div 1\frac{3}{5})$

5. 52% of $8\frac{3}{4}$

6. $\frac{5}{6}$% of 1,700

7. A certain organization requires that at least 45% of its members be women. If the organization has 65 members, what is the least number of women that can belong to it?

8. For every \$7,500 a business needs, you agree to supply \$6,000 of it.
   (a) What percent of the need are you supplying?
   (b) If the business needs \$40,000 how much must you supply?

9. You buy a radio that is being sold at 75% of its regular price. Later you sell the radio to a friend for 60% of the price you paid. If the regular price of the radio was \$140:
   (a) How much did you pay for it?
   (b) How much did your friend pay for it?
   (c) What percent of the regular price did your friend pay for the radio?

10. $4\frac{3}{4}$ pounds of apples cost \$3.61.
   (a) What is the cost per pound for the apples?
   (b) At this rate, how much would $14\frac{1}{4}$ pounds of apples cost?

ANSWERS:

1. _____
2. _____
3. (a) _____
   (b) _____
4. (a) _____
   (b) _____
5. _____
6. _____
7. _____
8. (a) _____
   (b) _____
9. (a) _____
   (b) _____
   (c) _____
10. (a) _____
    (b) _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 6, Form B

1. $15\frac{13}{30}$
2. $57\frac{13}{30}$
3. (a) $5\frac{7}{30}$ (b) $8\frac{13}{30}$
4. (a) $1\frac{71}{84}$ (b) $4\frac{76}{105}$
5. $4\frac{11}{20}$
6. $14\frac{1}{6}$
7. 30
8. (a) 80% (b) \$32,000
9. (a) \$105 (b) \$63 (c) 45%
10. (a) 76 cents (\$0.76) (b) \$10.83 (1,083 cents)

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If you did each problem in Self-Test 6, Form B correctly, you may, if you wish, proceed to the next module. Otherwise continue with Step 8.

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Step 8:

View the solutions for Self-Test 6, Form B on Videotape Lecture 6S. Pay special attention to the solutions of those problems for which you failed to get the correct answers. Rewind the tape at any time to restudy any problems that gave you difficulty.

Step 9:

Do Self-Test 6, Form C on the next page.

Self-Test 6, Form C

ANSWERS:

In exercises 1-6, write each answer as a mixed number.

1. $11\frac{2}{5} + 3\frac{1}{4} + 2\frac{2}{3}$ — 1. _____

2. $11\frac{2}{5} \times 3\frac{1}{4} \times 2\frac{2}{3}$ — 2. _____

3. (a) $(11\frac{2}{5} - 3\frac{1}{4}) - 2\frac{2}{3}$ — 3. (a) _____

   (b) $11\frac{2}{5} - (3\frac{1}{4} - 2\frac{2}{3})$ — (b) _____

4. (a) $(11\frac{2}{5} \div 3\frac{1}{4}) \div 2\frac{2}{3}$ — 4. (a) _____

   (b) $11\frac{2}{5} \div (3\frac{1}{4} \div 2\frac{2}{3})$ — (b) _____

5. 49% of $8\frac{3}{7}$ — 5. _____

6. $\frac{7}{11}$% of 2,000 — 6. _____

7. A certain organization requires that at least 51% of its members be women. If the organization has 90 members, what is the least number of women that can belong to it? — 7. _____

8. For every \$7,500 a business needs, you agree to supply \$5,000 of it. — 8. (a) _____

   (a) What percent of the need are you supplying?

   (b) If the business needs \$9,000 how much must you supply? — (b) _____

9. You buy a radio that is being sold at 60% of its regular price. Later you sell it to a friend for 70% of what you paid for it. If the regular price of the radio was \$200: — 9. (a) _____

   (a) How much did you pay for it? — (b) _____

   (b) How much did your friend pay for it? — (c) _____

   (c) What percent of the regular price did your friend pay for it?

10. $3\frac{2}{3}$ pounds of apples cost \$1.98. — 10. (a) _____

    (a) What is the cost of the apples per pound? — (b) _____

    (b) At this rate, how much would $7\frac{1}{3}$ pounds of apples cost?

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 6, Form C

1. $17\frac{19}{60}$
2. $98\frac{4}{5}$
3. (a) $5\frac{29}{60}$ (b) $10\frac{49}{60}$
4. (a) $1\frac{41}{130}$ (b) $9\frac{23}{65}$
5. $4\frac{13}{100}$
6. $12\frac{8}{11}$
7. 46
8. (a) $66\frac{2}{3}\%$ (b) $6,000
9. (a) $120 (b) $84 (c) 42%
10. (a) 54 cents ($0.54) (b) $3.96 (396 cents)

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THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE #6.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN MODULE #7.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE, YOU SHOULD CONSULT WITH A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.

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